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Renormalization of dimension 6 gluon operators

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ABSTRACT

We identify the independent dimension 6 twist 4 gluon operators and calculate their renormalization in the pure gauge theory. By constructing the renormalization group invariant combinations, we find the scale invariant condensates that can be estimated in nonperturbative calculations and used in QCD sum rules for heavy quark systems in medium.

1. Introduction

Understanding the changes of the matrix elements of the gluon operators near the critical temperature in QCD offers a useful picture on the nature of the QCD phase transition [1]. These can also be used in QCD sum rule analysis to understand the changes and melting of heavy quark system at finite temperature [2–4].

In the pure gauge theory, the lowest dimensional operators are the scalar gluon condensate and the twist 2 gluon operator. These dimension 4 operators can be reexpressed in terms of the electric condensate and the magnetic condensate. The temperature dependence of these operators can be calculated directly from lattice calculation of the space–time and space–space elementary plaquette [1,5] or from combining the calculation of the energy density and pressure [6]. The calculations show that while there is a rapid change of the electric condensate across the phase transition temperature, the magnetic condensate changes very little [6,7].

Using the temperature dependence of the dimension 4 condensates as the input in the QCD sum rule approach for the heavy quark system, J/ψ and η_c were found to undergo a rapid property change across the phase transition [3,6] and to their dissociation [4] slightly above the critical temperature. Moreover, it was also found that the free energy extracted from lattice calculation is the relevant potential to describe J/ψ in a potential model [8]. The extension to finite density also has interesting applications [9].

To further understand the phase transition in terms of local operators and to expand the findings for the charmonium system by using QCD sum rule to dimension 6 level, we will identify the dimension 6 and twist 4 gluon operators and calculate their renormalization in the pure gauge theory. The renormalization of scalar dimension 4 operators and scalar dimension 6 operators are well known [10,11]. Our result completes the calculation of renormalization of all the dimension 6 gluon operators, hence will be a first step toward identifying their mixing and thus a systematic analysis in the operator product expansion (OPE) of heavy quark correlation functions up to dimension 6 [12].

In Section 2, we will identify the independent operators at dimension 6. In Section 3, we will renormalize these independent operators up to one loop order. The scale invariant vacuum condensate will then be given in Section 4. Section 5 is a summary.

2. Independent operators

The gauge invariant dimension 6 operators are obtained by combining the covariant derivative D_μ and the field strength tensor $G_{\mu\nu}$. To find the independent even parity operators, we use the Bianchi identity and symmetry property of the indices. Here, we start from the operators that are of the type $(D_a G_{bc})(D_d G_{ef})$; that is, multiplication of two covariant components each composed of a covariant derivative acting on the field strength tensor.

For the scalar operator, the indices '*abcdef*' have to become '*aabbcc*' type. Considering the indices, the covariant component (DG) can have only two types. In the first case (*Type*1), the three indices '*abc*' are independent while in the second case (*Type*2), two indices are identical and summed over '*aab*'. Then, the scalar dimension 6 operator can be obtained from the product of the same type as follows.

$$(Type1)\times(Type1):\begin{pmatrix} D_aG_{bc}\\ D_bG_{ca}\\ D_cG_{ab}\end{pmatrix}\times\begin{pmatrix} D_aG_{bc}\\ D_bG_{ca}\\ D_cG_{ab}\end{pmatrix},$$
$$(Type2)\times(Type2):(\,D_aG_{ab}\,)\times(\,D_cG_{bc}\,)\,, \tag{1}$$

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where for both cases we have used the antisymmetry property of the field strength tensor to eliminate redundant scalar operators obtained with the two indices in the field strength tensor exchanged. Initially, nine operators can be constructed from $(Type1) \times (Type1)$, and one operator from $(Type2) \times (Type2)$. However, three kinds of covariant components of $(Type1)$ can be reduced to two independent components by the following Bianchi identity.

$$D_a G_{bc} + D_b G_{ca} + D_c G_{ab} = 0 \tag{2}$$

Therefore, four operators remain from $(Type1) \times (Type1)$. Furthermore, using the symmetry property of the indices and Bianchi identity, one can show that only one operator is independent, irrespective of the order of their indices. Specifically, we can express three of the remaining four operators in terms of the single operator $D_a G_{bc} D_a G_{bc}$,

$$D_a G_{bc} D_b G_{ca} = -1/2 D_a G_{bc} D_a G_{bc}$$
$$D_b G_{ca} D_a G_{bc} = -1/2 D_a G_{bc} D_a G_{bc}$$
$$D_b G_{ca} D_b G_{ca} = D_a G_{bc} D_a G_{bc}. \tag{3}$$

Therefore, there exist two independent scalar operators. That is,

$$D_\alpha G^a_{\mu\nu} D_\alpha G^a_{\mu\nu},\ D_\mu G^a_{\mu\alpha} D_\nu G^a_{\nu\alpha}. \tag{4}$$

Using the equation of motion, the second operator can be written in terms of quark operator, which vanishes in the pure gauge theory. Using higher dimensional Bianchi identity of the form $[D, [D, G]] = 0$, one can show that the usually quoted scalar operator can be obtained by combining the two independent operators.

$$g f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha} = D_\mu G^a_{\mu\alpha} D_\nu G^a_{\nu\alpha} - \frac{1}{2} D_\alpha G^a_{\mu\nu} D_\alpha G^a_{\mu\nu}. \tag{5}$$

Similarly, for the spin 2 operators, the indices '*abcdef*' become '*abccdd*'; that is, the external indices a, b remain free and symmetrized while c and d indices are summed over. Then, depending on how many external indices are included and on whether the remaining indices are summed over, there are four types of covariant components (DG) that become relevant.

$$Type1:\ D_a G_{bc},\quad D_b G_{ca},$$
$$Type2:\ D_a G_{cd},\quad D_c G_{da},\quad D_d G_{ac},$$
$$Type3:\ D_d G_{cd},$$
$$Type4:\ D_c G_{ac}, \tag{6}$$

where again we have used the antisymmetry property of the field strength tensor to eliminate redundant covariant components obtained with the two indices in the field strength tensor exchanged. Furthermore, one can reduce the number of independent covariant components in $(Type1)$ and $(Type2)$ by using the following Bianchi identities respectively.

$$D_a G_{bc} + D_b G_{ca} + D_c G_{ab} = D_a G_{bc} + D_b G_{ca} = 0, \tag{7}$$
$$D_a G_{cd} + D_c G_{da} + D_d G_{ac} = 0, \tag{8}$$

where the second equation in Eq. (7) follows for the present case as we will be looking spin two operators with symmetric external indices a and b. Thus, the spin 2 operators can be obtained by several combinations: one operator from $(Type1) \times (Type3)$, four operators from $(Type2) \times (Type2)$, and one operator from $(Type4) \times (Type4)$. Using the symmetry property of the indices and Bianchi identity similar to the scalar case, one can show that the four operators from $(Type2) \times (Type2)$ can be expressed in terms of two operators $D_a G_{cd} D_b G_{cd}$ and $D_c G_{da} D_c G_{db}$ only. Specifically, the remaining two from $(Type2) \times (Type2)$ can be expressed as

$$D_a G_{cd} D_c G_{db} = -1/2 D_a G_{cd} D_b G_{cd},$$
$$D_c G_{da} D_b G_{cd} = -1/2 D_a G_{cd} D_b G_{cd}. \tag{9}$$

Finally, using the higher dimensional Bianchi identity, one can obtain the following relation among the remaining four operators [12]:

$$D_c G_{da} D_c G_{db} = D_c G_{ac} D_d G_{bd} + D_a G_{cd} D_b G_{cd} + D_a G_{bc} D_d G_{cd}. \tag{10}$$

Hence, there are only three independent dimension 6 twist 4 gluon operators. In this work, we will use the following set [12]:

$$scalar:\ f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha},\ D_\mu G^a_{\mu\alpha} D_\nu G^a_{\nu\alpha}$$
$$spin2:\ D_\beta G^a_{\mu\nu} D_\alpha G^a_{\mu\nu},\ D_\mu G^a_{\alpha\mu} D_\nu G^a_{\beta\nu},\ D_\beta G^a_{\alpha\mu} D_\nu G^a_{\mu\nu}.$$

On the other hand, using the equation of motion, we find that only two gluon operators of dimension 6 remain in the pure gauge theory. These are $f^{abc} G^a_{\mu\nu} G^b_{\mu\alpha} G^c_{\nu\alpha}$ and $D_\beta G^a_{\mu\nu} D_\alpha G^a_{\mu\nu}$. The latter operator is proportional to $f^{abc} G^a_{\alpha\mu} G^b_{\beta\nu} G^c_{\mu\nu}$ with two spin indices $(\alpha\beta)$. Introducing the color E and B fields, we find the off diagonal components are of the forms $E^a_\parallel B^b_\perp B^c_\parallel$ or $E^a_\parallel E^b_\perp B^c_\parallel$, the matrix elements of which vanish in the medium at rest due to rotational invariance. Therefore, the two independent dimension 6 operators in the pure gauge theory that remain and that constitute the diagonal components and the scalar operators are $f^{abc} B^a \cdot (B^b \times B^c)$ and $f^{abc} B^a \cdot (E^b \times E^c)$.

3. Renormalization

The renormalization of scalar operators are reported in Ref. [10]. Here, we will focus on the twist 4(spin2 traceless) part. We will use the three independent set as mentioned in the previous section and therefore discuss the renormalization of the following three operators:

$$O_1 = D_\beta G^a_{\mu\nu} D_\alpha G^a_{\mu\nu}|_{ST}, \tag{11}$$
$$O_2 = D_\mu G^a_{\alpha\mu} D_\nu G^a_{\beta\nu}|_{ST}, \tag{12}$$
$$O_3 = D_\beta G^a_{\alpha\mu} D_\nu G^a_{\mu\nu}|_{ST}, \tag{13}$$

where we have suppressed the external indices α, β in the left hand side and used the notation $O_{\alpha\beta}|_{ST}$ which means $1/2(O_{\alpha\beta} + O_{\beta\alpha}) - 1/4 g_{\alpha\beta}(O^\mu_\mu)$ making the operators twist 4. First, we will study the renormalization of O_1 up to one loop order using the background field method with zero momentum insertion [13].

To study the renormalization of the first operator, we consider the following Green's functions with external fields,

$$\langle A^a_\mu A^b_\nu A^c_\lambda O_1 \rangle = Z_{1,1} Z_A \langle A^a_\mu A^b_\nu A^c_\lambda O_{1B} \rangle + \sum_{j=2}^{3} Z_{1,j} \langle A^a_\mu A^b_\nu A^c_\lambda O_{jB} \rangle. \tag{14}$$

Here, A^a_μ is the background gluon field and Z_A the background field renormalization constant. O_{1B} represents the bare operator with renormalized fields and coupling.

The diagrams that contribute to the renormalization are shown in Fig. 1 with the Feynman rules given in Fig. 2. The two gluon vertex comes from contraction with $A^a_\mu(p) A^b_\nu(q)$, the three gluon vertex with $A^a_\mu(p) A^b_\nu(q) A^c_\lambda(r)$, and the four gluon vertex with $A^a_\mu(p) A^b_\nu(q) A^c_\lambda(r) A^d_\omega(k)$. For each vertex, there are $N_G!$ ways of

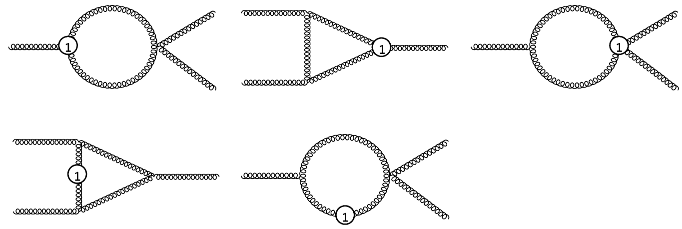
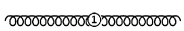
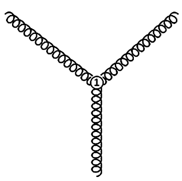


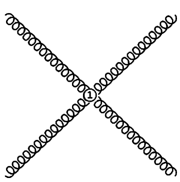
Fig. 1. Diagrams contributing to the renormalization of O_1 to one loop order in the pure gauge theory.



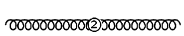
$$4p^\alpha p^\beta \delta_{ab}(p^2 g^{\mu\nu} - p^\mu p^\nu) - \frac{1}{4} g^{\alpha\beta}(4p^4 \delta_{ab} g^{\mu\nu} - 4p^2 p^\mu p^\nu \delta_{ab})$$



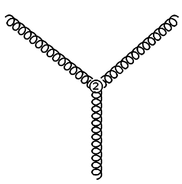
$$-\frac{1}{2} igf_{abc}(2p^\alpha p^\lambda q^\beta g^{\mu\nu} - 2p^\alpha p^\nu q^\beta g^{\lambda\mu} + 2p^\beta q^\alpha (p^\lambda g^{\mu\nu} - p^\nu g^{\lambda\mu}) - p^\lambda g^{\alpha\beta} g^{\mu\nu} p\cdot q + p^\nu g^{\alpha\beta} g^{\lambda\mu} p\cdot q + 2p^\alpha p^\lambda r^\beta g^{\mu\nu}$$
$$+ 4p^\alpha p^\lambda r^\mu g^{\beta\nu} + 4p^\beta p^\lambda r^\mu g^{\alpha\nu} - 2p^\alpha p^\nu r^\beta g^{\lambda\mu} - 2p^\lambda p^\nu r^\mu g^{\alpha\beta} + 2p^\beta r^\alpha (p^\lambda g^{\mu\nu} - p^\nu g^{\lambda\mu}) - p^\lambda g^{\alpha\beta} g^{\mu\nu} p\cdot r - 4p^\alpha g^{\beta\nu} g^{\lambda\mu} p\cdot r$$
$$- 4p^\beta g^{\alpha\nu} g^{\lambda\mu} p\cdot r + 3p^\nu g^{\alpha\beta} g^{\lambda\mu} p\cdot r) + (\text{5 other terms from permutation of contraction order})$$



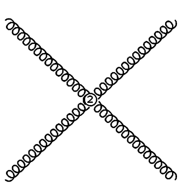
$$-\frac{1}{4} g^2 f_{abx} f_{cdx}(4k^\alpha p^\beta g^{\lambda\mu} g^{\nu\omega} + 4r^\alpha p^\beta g^{\lambda\mu} g^{\nu\omega} + 4k^\alpha q^\beta g^{\lambda\mu} g^{\nu\omega} + 4r^\alpha q^\beta g^{\lambda\mu} g^{\nu\omega} + 4k^\alpha g^{\beta\mu} q^\lambda g^{\nu\omega} + 4r^\alpha g^{\beta\mu} q^\lambda g^{\nu\omega}$$
$$+ 4g^{\alpha\lambda} p^\beta k^\mu g^{\nu\omega} 4g^{\alpha\lambda} q^\beta k^\mu g^{\nu\omega} - g^{\alpha\beta} p^\lambda k^\mu g^{\nu\omega} - 2g^{\alpha\beta} q^\lambda k^\mu g^{\nu\omega} - g^{\alpha\beta} q^\lambda r^\mu g^{\nu\omega} - g^{\alpha\beta} g^{\lambda\mu} k\cdot p g^{\nu\omega} + 8g^{\alpha\lambda} g^{\beta\mu} k\cdot q g^{\nu\omega}$$
$$- 3g^{\alpha\beta} g^{\lambda\mu} k\cdot q g^{\nu\omega} - g^{\alpha\beta} g^{\lambda\mu} p\cdot r g^{\nu\omega} - g^{\alpha\beta} g^{\lambda\mu} q\cdot r g^{\nu\omega} - 4g^{\alpha\lambda} p^\beta g^{\mu\omega} k^\nu - 4g^{\alpha\lambda} q^\beta g^{\mu\omega} k^\nu + g^{\alpha\beta} p^\lambda g^{\mu\omega} k^\nu$$
$$+ g^{\alpha\beta} q^\lambda g^{\mu\omega} k^\nu - 2g^{\alpha\beta} p^\lambda g^{\mu\omega} p^\nu + 2g^{\alpha\beta} g^{\lambda\mu} p^\nu p^\omega + 4p^\alpha g^{\beta\nu}(p^\lambda g^{\mu\omega} - g^{\lambda\mu} p^\omega) + 4g^{\alpha\nu} p^\beta (p^\lambda g^{\mu\omega} - g^{\lambda\mu} p^\omega)$$
$$- 4k^\alpha g^{\beta\mu} g^{\lambda\nu} q^\omega - 4r^\alpha g^{\beta\mu} g^{\lambda\nu} q^\omega + g^{\alpha\beta} g^{\lambda\nu} k^\mu q^\omega + g^{\alpha\beta} g^{\lambda\nu} r^\mu q^\omega - 8g^{\alpha\lambda} g^{\beta\mu} k^\nu q^\omega + 2g^{\alpha\beta} g^{\lambda\mu} k^\nu q^\omega) + (\text{23 other terms})$$



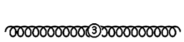
$$\frac{1}{2}\delta_{ab}(p^2(g^{\alpha\nu}(2p^2 g^{\beta\mu} - 2p^\beta p^\mu) + g^{\alpha\mu}(2p^2 g^{\beta\nu} - 2p^\beta p^\nu) + g^{\alpha\beta}(p^\mu p^\nu - p^2 g^{\mu\nu})) - 2p^\alpha(p^2(p^\mu g^{\beta\nu} + p^\nu g^{\beta\mu}) - 2p^\beta p^\mu p^\nu))$$



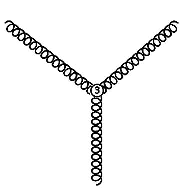
$$\frac{1}{2} igf_{abc}(2p^\beta p^\mu q^\lambda g^{\alpha\nu} - p^\mu p^\nu q^\lambda g^{\alpha\beta} - 2p^2 q^\lambda g^{\alpha\nu} g^{\beta\mu} - 2p^2 q^\lambda g^{\alpha\mu} g^{\beta\nu} + p^2 q^\lambda g^{\alpha\beta} g^{\mu\nu} + 2p^\alpha p^\mu (g^{\beta\nu}(q^\lambda + r^\lambda) + r^\beta g^{\lambda\nu}$$
$$- r^\nu g^{\beta\lambda}) + 2p^\beta p^\mu r^\lambda g^{\alpha\nu} - p^\mu p^\nu r^\lambda g^{\alpha\beta} - 2p^\beta p^\mu r^\nu g^{\alpha\lambda} + p^\lambda p^\mu r^\nu g^{\alpha\beta} - 2p^2 r^\beta g^{\alpha\mu} g^{\lambda\nu} - 2p^2 r^\lambda g^{\alpha\nu} g^{\beta\mu} - 2p^2 r^\lambda g^{\alpha\mu} g^{\beta\nu}$$
$$+ p^2 r^\lambda g^{\alpha\beta} g^{\mu\nu} + p^2 r^\mu g^{\alpha\beta} g^{\lambda\nu} + 2p^2 r^\nu g^{\alpha\mu} g^{\beta\lambda} + 2p^2 r^\nu g^{\alpha\lambda} g^{\beta\mu} - p^2 r^\nu g^{\alpha\beta} g^{\lambda\mu} + 2r^\alpha g^{\lambda\nu}(p^\beta p^\mu - p^2 g^{\beta\mu}) - p^\mu g^{\alpha\beta} g^{\lambda\nu} p\cdot r)$$
$$+ (\text{5 other terms})$$



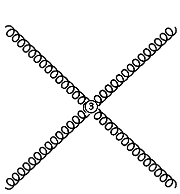
$$-\frac{1}{4} g^2 f_{abx} f_{cdx}(4p^\alpha g^{\beta\lambda} p^\mu g^{\nu\omega} + 4g^{\alpha\lambda} p^\beta p^\mu g^{\nu\omega} - 2g^{\alpha\beta} p^\lambda p^\mu g^{\nu\omega} - g^{\alpha\beta} k^\lambda q^\mu g^{\nu\omega} - 4g^{\alpha\mu} g^{\beta\lambda} p^2 g^{\nu\omega} - 4g^{\alpha\lambda} g^{\beta\mu} p^2 g^{\nu\omega}$$
$$+ 2g^{\alpha\beta} g^{\lambda\mu} p^2 g^{\nu\omega} + g^{\alpha\beta} g^{\lambda\omega} q^\mu k^\nu + 4g^{\alpha\mu} k^\beta g^{\lambda\omega} p^\nu - 4g^{\alpha\mu} g^{\beta\omega} k^\lambda p^\nu + g^{\alpha\beta} k^\lambda g^{\mu\omega} p^\nu - g^{\alpha\beta} g^{\lambda\omega} k^\mu p^\nu + 4g^{\alpha\mu} k^\beta g^{\lambda\omega} q^\nu$$
$$- 4g^{\alpha\mu} g^{\beta\omega} k^\lambda q^\nu + g^{\alpha\beta} k^\lambda g^{\mu\omega} q^\nu - g^{\alpha\beta} g^{\lambda\omega} k^\mu q^\nu - g^{\alpha\beta} q^\lambda g^{\mu\nu} k^\omega + g^{\alpha\beta} g^{\lambda\nu} q^\mu k^\omega + 4g^{\alpha\mu} g^{\beta\lambda} p^\nu k^\omega - g^{\alpha\beta} g^{\lambda\mu} p^\nu k^\omega$$
$$+ 4g^{\alpha\mu} g^{\beta\lambda} q^\nu k^\omega - g^{\alpha\beta} g^{\lambda\mu} q^\nu k^\omega + g^{\alpha\beta} k^\lambda g^{\mu\nu} q^\omega - g^{\alpha\beta} q^\lambda g^{\mu\nu} r^\omega + g^{\alpha\beta} g^{\lambda\nu} q^\mu r^\omega + 4g^{\alpha\mu} g^{\beta\lambda} p^\nu r^\omega - g^{\alpha\beta} g^{\lambda\mu} p^\nu r^\omega$$
$$+ 4g^{\alpha\mu} g^{\beta\lambda} q^\nu r^\omega - g^{\alpha\beta} g^{\lambda\mu} q^\nu r^\omega + 4q^\alpha g^{\mu\nu}(k^\beta g^{\lambda\omega} - g^{\beta\omega} k^\lambda + g^{\beta\lambda}(k^\omega + r^\omega)) - 4g^{\alpha\nu} q^\mu (k^\beta g^{\lambda\omega} - g^{\beta\omega} k^\lambda$$
$$+ g^{\beta\lambda}(k^\omega + r^\omega)) - g^{\alpha\beta} g^{\lambda\omega} g^{\mu\nu} k\cdot q) + (\text{23 other terms})$$



$$\frac{1}{2}\delta_{ab}(4p^\alpha p^\beta - p^2 g^{\alpha\beta})(p^\mu p^\nu - p^2 g^{\mu\nu})$$



$$\frac{1}{4} igf_{abc}(4r^\alpha g^{\beta\nu} p^\lambda p^\mu + 2g^{\alpha\nu} q^\beta p^\lambda p^\mu + 4g^{\alpha\nu} r^\beta p^\lambda p^\mu + g^{\alpha\beta} q^\lambda p^\nu p^\mu + g^{\alpha\beta} r^\lambda p^\nu p^\mu - g^{\alpha\beta} p^\lambda q^\nu p^\mu - 3g^{\alpha\beta} p^\lambda r^\nu p^\mu$$
$$- 2g^{\alpha\nu} g^{\beta\lambda} p\cdot r p^\mu - 2g^{\alpha\lambda} g^{\beta\nu} p\cdot r p^\mu + 2g^{\alpha\beta} g^{\lambda\nu} p\cdot r p^\mu - 2g^{\alpha\mu} p^\beta q^\lambda p^\nu - 2g^{\alpha\mu} p^\beta r^\lambda p^\nu + 2g^{\alpha\mu} p^\beta p^\lambda r^\nu - 4r^\alpha g^{\beta\nu} g^{\lambda\mu} p^2$$
$$- 2g^{\alpha\nu} q^\beta g^{\lambda\mu} p^2 - 4g^{\alpha\nu} r^\beta g^{\lambda\mu} p^2 - g^{\alpha\beta} q^\lambda g^{\mu\nu} p^2 - g^{\alpha\beta} r^\lambda g^{\mu\nu} p^2 + 2g^{\alpha\nu} g^{\beta\lambda} r^\mu p^2 + 2g^{\alpha\lambda} g^{\beta\nu} r^\mu p^2 - 2g^{\alpha\beta} g^{\lambda\nu} r^\mu p^2$$
$$+ g^{\alpha\beta} g^{\lambda\mu} q^\nu p^2 + 3g^{\alpha\beta} g^{\lambda\mu} r^\nu p^2 + 2q^\alpha g^{\beta\nu}(p^\lambda p^\mu - g^{\lambda\mu} p^2) - 2g^{\alpha\mu} p^\beta g^{\lambda\nu} p\cdot r + 2p^\alpha (2p^\beta (q^\lambda g^{\mu\nu} + r^\lambda g^{\mu\nu} + g^{\lambda\nu} r^\mu - g^{\lambda\mu} r^\nu)$$
$$- g^{\beta\mu}(q^\lambda p^\nu + r^\lambda p^\nu - p^\lambda r^\nu + g^{\lambda\nu} p\cdot r))) + (\text{5 other terms})$$



$$\frac{1}{4} g^2 f_{abx} f_{cdx}(2g^{\alpha\mu} p^\beta k^\lambda g^{\nu\omega} + 4g^{\alpha\mu} q^\beta k^\lambda g^{\nu\omega} + 2g^{\alpha\mu} p^\beta p^\lambda g^{\nu\omega} - g^{\alpha\beta} k^\lambda p^\mu g^{\nu\omega} - g^{\alpha\beta} p^\lambda p^\mu g^{\nu\omega} - 2g^{\alpha\beta} k^\lambda q^\mu g^{\nu\omega}$$
$$+ g^{\alpha\beta} g^{\lambda\mu} p^2 g^{\nu\omega} - 2g^{\alpha\mu} p^\beta g^{\lambda\omega} k^\nu - 4g^{\alpha\mu} q^\beta g^{\lambda\omega} k^\nu + g^{\alpha\beta} g^{\lambda\omega} p^\mu k^\nu + 2g^{\alpha\beta} g^{\lambda\omega} q^\mu k^\nu - 2g^{\alpha\mu} p^\beta g^{\lambda\nu} k^\omega - 4g^{\alpha\mu} q^\beta g^{\lambda\nu} k^\omega$$
$$+ 2g^{\alpha\nu} g^{\beta\mu} q^\lambda k^\omega + 2g^{\alpha\mu} g^{\beta\nu} q^\lambda k^\omega - g^{\alpha\beta} q^\lambda g^{\mu\nu} k^\omega + g^{\alpha\beta} g^{\lambda\nu} p^\mu k^\omega + 2g^{\alpha\beta} g^{\lambda\nu} q^\mu k^\omega - 2g^{\alpha\nu} g^{\beta\lambda} p^\mu p^\omega - 2g^{\alpha\lambda} g^{\beta\nu} p^\mu p^\omega$$
$$+ g^{\alpha\beta} g^{\lambda\nu} p^\mu p^\omega - 2g^{\alpha\nu} g^{\beta\mu} k^\lambda q^\omega - 2g^{\alpha\mu} g^{\beta\nu} k^\lambda q^\omega + g^{\alpha\beta} k^\lambda g^{\mu\nu} q^\omega - 2g^{\alpha\mu} p^\beta g^{\lambda\nu} r^\omega - 4g^{\alpha\mu} q^\beta g^{\lambda\nu} r^\omega + 2g^{\alpha\nu} g^{\beta\mu} q^\lambda r^\omega$$
$$+ 2g^{\alpha\mu} g^{\beta\nu} q^\lambda r^\omega - g^{\alpha\beta} q^\lambda g^{\mu\nu} r^\omega + g^{\alpha\beta} g^{\lambda\nu} p^\mu r^\omega + 2g^{\alpha\beta} g^{\lambda\nu} q^\mu r^\omega + 4q^\alpha g^{\beta\mu}(k^\lambda g^{\nu\omega} - g^{\lambda\omega} k^\nu - g^{\lambda\nu}(k^\omega + r^\omega))$$
$$- 2p^\alpha (2p^\beta g^{\lambda\mu} g^{\nu\omega} + g^{\beta\mu}(-k^\lambda g^{\nu\omega} - p^\lambda g^{\nu\omega} + g^{\lambda\omega} k^\nu + g^{\lambda\nu} k^\omega + g^{\lambda\nu} r^\omega)) + 2g^{\alpha\nu} g^{\beta\mu} g^{\lambda\omega} k\cdot q + 2g^{\alpha\mu} g^{\beta\nu} g^{\lambda\omega} k\cdot q$$
$$- g^{\alpha\beta} g^{\lambda\omega} g^{\mu\nu} k\cdot q + 2g^{\alpha\nu} g^{\beta\lambda} g^{\mu\omega} p^2 + 2g^{\alpha\lambda} g^{\beta\nu} g^{\mu\omega} p^2 - g^{\alpha\beta} g^{\lambda\nu} g^{\mu\omega} p^2) + (\text{23 other terms})$$

Fig. 2. Feynman rules in the background field method for the pure gauge theory.

contraction, where N_G is the number of external gluon fields. In Fig. 2, we just show one contracted component for three and four gluon vertex. Other remaining components can be obtained by permutation of $\{p, \mu, a\}\{q, \nu, b\}\{r, \lambda, c\}$ for three gluon and $\{p, \mu, a\}\{q, \nu, b\}\{r, \lambda, c\}\{k, \omega, d\}$ for four gluon vertex.

The calculation is performed using dimensional regularization $D = 4 - 2\epsilon$ in Feynman gauge for SU(N). The calculation for the other operators involves the same diagrams and hence can be repeated similarly. The renormalization terms for each operator was indeed found to be observed into a linear combination of the independent operators, which is a nontrivial check of the calculations. The following is the collected result of the renormalization constants.

$$Z_{1,1} = 1 + \frac{3N}{4}\frac{\alpha_s}{\pi\epsilon} \tag{15}$$

$$Z_{1,2} = -\frac{N}{12}\frac{\alpha_s}{\pi\epsilon} \tag{16}$$

$$Z_{1,3} = \frac{2N}{3}\frac{\alpha_s}{\pi\epsilon} \tag{17}$$

$$Z_{2,1} = 0 \tag{18}$$

$$Z_{2,2} = 1 + \frac{N}{3}\frac{\alpha_s}{\pi\epsilon} \tag{19}$$

$$Z_{2,3} = \frac{N}{24}\frac{\alpha_s}{\pi\epsilon} \tag{20}$$

$$Z_{3,1} = 0 \tag{21}$$

$$Z_{3,2} = \frac{N}{6}\frac{\alpha_s}{\pi\epsilon} \tag{22}$$

$$Z_{3,3} = 1 + \frac{7N}{24}\frac{\alpha_s}{\pi\epsilon}. \tag{23}$$

4. Scale invariant condensates

The scale invariant condensates can be obtained by diagonalization the following matrix Z.

$$Z = \begin{pmatrix} 1 + \frac{3N\alpha_s}{4\pi\epsilon} & -\frac{N\alpha_s}{12\pi\epsilon} & \frac{2N\alpha_s}{3\pi\epsilon} \\ 0 & 1 + \frac{N\alpha_s}{3\pi\epsilon} & \frac{N\alpha_s}{24\pi\epsilon} \\ 0 & \frac{N\alpha_s}{6\pi\epsilon} & 1 + \frac{7N\alpha_s}{24\pi\epsilon} \end{pmatrix}. \tag{24}$$

We then find the following new operator set, which corresponds to the eigenvectors of Z.

$$\langle O_{1\text{new}} \rangle = \langle O_1 \rangle \tag{25}$$

$$\langle O_{2\text{new}} \rangle = \left\langle \frac{-653 + 21\sqrt{17}}{424} O_1 + \frac{1 - \sqrt{17}}{8} O_2 + O_3 \right\rangle \tag{26}$$

$$\langle O_{3\text{new}} \rangle = \left\langle \frac{-653 - 21\sqrt{17}}{424} O_1 + \frac{1 + \sqrt{17}}{8} O_2 + O_3 \right\rangle. \tag{27}$$

These are renormalized multiplicatively without mixing. The renormalization constants correspond to the eigenvalues of Z.

$$\langle O_{1\text{new}} \rangle = \left(1 + \frac{3N\alpha_s}{4\pi\epsilon}\right) \left\langle O^0_{1\text{newB}} \right\rangle \tag{28}$$

$$\langle O_{2\text{new}} \rangle = \left(1 + \frac{\left(15 - \sqrt{17}\right) N\alpha_s}{48\pi\epsilon}\right) \left\langle O^0_{2\text{newB}} \right\rangle \tag{29}$$

$$\langle O_{3\text{new}} \rangle = \left(1 + \frac{\left(15 + \sqrt{17}\right) N\alpha_s}{48\pi\epsilon}\right) \left\langle O^0_{3\text{newB}} \right\rangle. \tag{30}$$

O^0_{newB} means bare operator with bare fields and coupling. Finally, we can obtain the scale invariant condensates at the one loop order by multiplying these operators with corresponding factors of the coupling α_s so that the renormalization of the coupling cancels that of the operator [11].

$$\phi_1 = \alpha_s^{-\frac{9}{11}} \langle O_{1\text{new}} \rangle \tag{31}$$

$$\phi_2 = \alpha_s^{-\frac{15-\sqrt{17}}{44}} \langle O_{2\text{new}} \rangle \tag{32}$$

$$\phi_3 = \alpha_s^{-\frac{15+\sqrt{17}}{44}} \langle O_{3\text{new}} \rangle. \tag{33}$$

5. Summary

We have identified and calculated the renormalization of the dimension 6 twist 4 gluon operators to one loop order in the pure gauge theory. Among the three independent operators, O_1 was found to not mix with other operators O_2 and O_3, which vanishes in the pure gauge theory. Moreover, as can be seen in Eq. (7), O_1 is related to the second moment of the dimension 4 scalar gluon operator as it is composed of two covariant derivatives with symmetric and traceless indices acting between the scalar gluon operator. Hence, O_1 could be the first operator that can be estimated in a nonperturbative model or calculated on the lattice. With our calculation, the renormalization of all the dimension 6 operators are now known. The QCD sum rule methods for the heavy quark system in medium can now be systematically studied up to dimension 6 level.

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